Teaching through Problem Solving

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APPENDIX: TEACHING MOVES FOR TtPS

The use of tasks within pedagogies of sense making and problem solving can adhere to the following general teaching moves. The roles of the teacher and students are embedded in these moves.

Move 1
- Pose the whole task; facilitate students’ understanding of the problem. Provide the space and opportunity for students to explore the mathematics by “posing problems that are within students’ reach, allowing them to struggle to find solutions and then examining the methods they have used” (Hiebert and Wearne 2003, p. 6).
- “Tasks must allow students to treat the situation as problematic, as something they need to think about rather than a prescription they need to follow. Secondly, what is problematic about the task should be the mathematics rather than other aspects of the situation. Finally, in order for students to work seriously on a task, it must offer students the chance to use skills and knowledge they already possess” (Hiebert, Carpenter, et al. 1997, p. 18).
- A caveat: Do not remove the mathematical complexities of the problem. Do not break down the problem so that students will not have to struggle productively to generate understanding.

Move 2
- Think Phase: Allow time for students to explore the messiness of the problem, generate conjectures, and build-understanding. This phase is akin to Polya’s (1945) “devise a plan” and “carry out the plan” approach or Mason, Burton, and Stacey’s (1985) notion of attack. Students are engaged in solving and resolving problems. The teacher should observe and take notes of students’ strategies, their difficulties, and what they are able to do. The idea is not to provide hints or feedback that remove the mathematical complexity, mathematical struggle, or the intended cognitive engagement (Stein at al. 2000; Garden et al. 2006).
- Share Phase: The students make public their ideas, methods, plans, and results. An alternative step is the Think-Pair-Share Phase: The teaching move could also incorporate a microunction of sharing in smaller groups before the whole-class share phase. As students share, the teacher should monitor the mathematical language and accuracy of the mathematics to help sequence the whole-class share phase. In the whole-class share phase, the teacher should consider sequencing students’ work, beginning with the intuitive-concrete approaches and strategies and moving to the more abstract approaches. The starting point matters, in the lens of the Equity Principle (NCTM 2000), because a starting point that allows all students to enter into the discussion of the mathematics affords students better opportunities to engage with the mathematical learning and its progression. This notion of progressive formalization (Bransford, Brown, and Cocking 2000) is a most important component of TtPS. We call the aforementioned process an equitable practice that necessarily provides access, support, and enrichment all at once.

Move 3
- Students and the teacher discuss the mathematics through mathematical practices:
  - Focus on concepts and skills
    - Focus on a big idea of the mathematics, make it explicit and public, and reason about and from it.
  - Focus on methods
    - Examine increasingly better solution methods (Hiebert and Wearne 2003, pp. 5–6).
    - Progressively formalize the mathematics from students’ concrete strategies to abstract strategies. If students do not discover a strategy of value that can bridge current knowledge and new learning, then the teacher has the right to introduce that strategy skillfully to move students forward.
    - The teacher should present alternative methods to resolve the mathematical problem or task—those that “have not been suggested by students” (Hiebert and Wearne 2003, p. 11)
    - Emphasize and label the efficient method as a capstone to exploration.
○ Highlight information and structures in different methods and strategies.
○ Highlight connections among different methods and strategies.
○ Help students focus on what is important mathematically (i.e., what is the mathematical residue? See Hiebert, Carpenter, et al. 1997). Focus on structure: What are the big containers into which these ruminations can be housed?

(c) Highlight essential mathematical ideas (might require heavy teacher involvement through questioning and strategic telling)

(d) Build skill fluency (or symbolic intuition, as Clemens has specified [2008]) and conceptual understanding. Symbolic intuition can be defined as knowledge of procedures, how to use the procedures, and using the procedures efficiently. An example of symbolic intuition could be the ability to effectively use mathematical induction in a proving task when it is needed; knowing about induction is not enough, and the ability to effectively use induction is actually cognitively and mathematically complex.

**Move 4**

- Make ideas visible by capturing and recording individual and group contributions on a public space for public viewing:

(a) Focus on using mistakes to unpack difficulties, misconceptions, and reasoning. Support interactivity that encourages students to step out of their comfort zones in the knowledge that making mistakes and learning from such mistakes are parts of the learning process (Shulman 2005).

(b) Focus on facilitating accountability (Shulman 2005).

(c) Focus on reason, not on people or personalities.

**Move 5**

- Closure: Mathematical understanding develops over time through articulated and coherent “mathematical struggle” negotiated in small steps:

(a) Provide time for individual and private reflections on what was learned at the end of the sequence of progressively formalized and connected learning episodes. Part of this can be assignments to help students reinforce, elaborate, and extend understanding.

(b) Students and teacher should make public and explicit the intended mathematical residue (Hiebert and Grouws 2007).

**BIBLIOGRAPHY**


