Reasoning and sense making should occur in every classroom every day,” states Focus in High School Mathematics: Reasoning and Sense Making (NCTM 2009, p. 5). As this book suggests, reasoning can take many forms, including explorations and conjectures as well as explanations and justifications of student thinking. Sense making, on the other hand, is “developing understanding of a situation, context, or concept by connecting it with existing knowledge” (NCTM 2009, p. 4). Classroom assessment can provide opportunities for students to share and enhance their mathematical reasoning and sense making, allowing student reasoning and sense making to become visible to students and teachers so that they can respond in appropriate ways. However, what does such assessment actually look like in classrooms? This article draws on a multiyear research project of grades 7–10 mathematics teachers to describe assessment practices that support students’ mathematical reasoning and sense making.

Current thinking and research in classroom assessment promote the use of assessment to support student learning. This is a move away from assessment as merely an event occurring at the end of a unit to provide a mark for reporting and a move toward assessment as a process of ongoing feedback. Such ongoing assessment is referred to variously. I use the term formative assessment to refer to the gathering and use of information about students’ learning that helps both teachers and students modify teaching and learning activities. Evidence that formative assessment is a powerful lever for improving student learning has been steadily accumulating over the last quarter of a century (Black and Wiliam 1998; Brookhart 2004, 2007; Natriello 1987; Wiliam 2007). In fact, compelling research results indicate that the practice of formative assessment may be the most significant single factor in raising the academic achievement of all students—and especially that of lower-achieving students (Black and Wiliam 1998; Brookhart 2007).

With respect to mathematics, formative assessment provides teachers with a window into students’ mathematical reasoning and sense making and creates a forum for student and teacher discussion about such thinking (Gearhart and Saxe...
Formative assessment recognizes that mathematical problem solving is a complex task that cannot be measured by merely a paper-and-pencil test; rather, assessment takes on a variety of forms, such as observations, conferencing, questioning, quizzes, progress reports, journal entries, and submission of problem-solving work (Romagnano 2001; Wiliam 2007). Thus, formative assessment prompts the use of a variety of types of assessment to understand and encourage mathematical reasoning and sense making and to guide teachers in their instructional actions. Such assessment also helps create a classroom culture of respect and success (Brookhart 2001; Shepard 2000).

The data from the Curriculum Implementation in Intermediate Math (CIIM) research project help provide some sense of teachers’ use of formative assessment and examples of what such assessment might look like in a classroom. The project examined the implementation of a provincewide move toward an inquiry approach to mathematics in Ontario, Canada (Suurtamm and Graves 2007). It examined a variety of teacher practices, beliefs, supports, and challenges through teacher questionnaires ($n = 1096$); analysis of provincial curriculum documents and resources to support the curriculum; focus groups with teachers and leaders in mathematics; and nine case studies of teachers’ classroom practice.

What did the data tell us about teachers’ assessment practices? A summary of the analysis of the questionnaire data provides an overview of teacher practices. The data from the teacher questionnaire suggest that although many Ontario teachers rely heavily on tests and quizzes, they also use other assessment strategies. The data also suggest that teachers use a greater variety of strategies to get a sense of students’ understanding than to determine a grade (Suurtamm, Koch, and Arden 2010). These strategies include performance tasks, conferencing, projects, interviews, observation, and responses of students in class.

Whereas the questionnaire data show that some teachers are incorporating innovative assessment ideas, the case studies provide details of what these look like in classrooms. The classrooms for the case studies were chosen because they promoted mathematical inquiry, investigation, and problem solving.
For each case study, we observed and videotaped four to six mathematics lessons and conducted interviews with the classroom teacher as well as school or department administrators. Examples from three case studies, one each from grades 8, 9 and 10, are presented here to describe assessment practices that supported students’ mathematical reasoning and sense making.

**CASE STUDY 1: MATH FORUM**

Angela is a grade 8 teacher who has established a routine that she calls Math Forum, which she adapted from Fosnot’s work (e.g., Fosnot and Dolk 2001). A Math Forum begins with Angela presenting a problem such as this one to her students:

In the first tug of war, 4 frogs on one side had a tie with 5 fairy godmothers on the other side. In the second tug of war, 1 dragon had a tie with 2 fairy godmothers and 1 frog. The third tug of war was between 1 dragon and 3 fairy godmothers on one side and 4 frogs on the other side. Who won the tug of war?

Angela’s rationale for choosing this problem was that it allowed students to use a variety of strategies. Also, she felt that the discussion and comparison of strategies would highlight algebraic reasoning through concepts such as equivalence and substitution used in the various strategies.

After Angela presented the problem, students worked in pairs, writing their ideas and strategies on large sheets of paper. Students had access to a variety of materials, such as linking cubes and two-color counters, and used these to construct mathematical models, make conjectures, and connect their ideas. As students worked, Angela walked around the room, validating and building on students’ work while questioning them to develop their ideas and concepts further. She asked student pairs such questions as “Can you elaborate?” or made such suggestions as “This makes sense when you explain it to me. Can you write down your thinking?”

A discussion with one pair of students, Jason and Mike, follows:

**Angela:** So, Jason, can you explain what you did?

**Jason:** Well, 4 frogs and 5 fairy godmothers have the same strength. So we divided 4 by 5 to get the strength of 1 frog.

**Angela:** What made you think of unit rates?

**Jason:** Because we wanted to get some numbers to compare them.

**Angela:** Tell me again why you divided by 5?

**Mike:** Well, we needed some way to get things equal.

**Jason:** Yeah, so now we have a way to substitute frogs for fairies.

As she observed students working, Angela recorded on a form, next to each student’s name, the types of strategies and representations each used. At the end of the forty-minute period, she told the class that the next day some student pairs would present their solutions to the class. In preparation, she reviewed her recorded observations and students’ work to determine the pairs that would present and the sequence of the presentations so that the class would see a range of strategies and could connect different representations of similar strategies.

The next class began with the presentation of solutions; students who did not present paraphrased the solutions and asked questions of the presenters. Several pairs had used proportional reasoning. For instance, Pam and Brad used a concept from Pokémon™ and gave the creatures in the story different fighting strengths. They chose the number 20 and “weighted” the strengths of frogs as 5 and godmothers as 4 to balance the first equation. They calculated the strength of the dragon using the second equation and substituted that in the third equation to determine the direction of the inequality (see fig. 1).

Some other student pairs also used proportional reasoning in their solutions. For example, Mike and Jason worked with unit rates. Addressing the class, Angela commented, “I’m confused. Why does this work? One group used different numbers than the other group.” The class then discussed the concept of proportional reasoning, and one student explained, “It doesn’t matter what the numbers are as long as the ratios are the same.”

Several other student pairs used different forms of substitution. Josh and Sara used the reasoning shown in figure 2, and the class discussed this form of substitution.

Angela then gave students time to try some of the strategies that had been presented. Many used a variation of Josh and Sara’s substitution. Some used symbols such as $F$, $G$, and $D$ to represent the
characters; thus, the first round was represented by the equation $4F = 5G$. The class concluded with a discussion of the different ways in which substitution was used and the role of the equals sign as signifying a balance of both sides.

In her interview, Angela stated that Math Forum provides her with multiple assessment opportunities: observing students’ collaborative problem solving; reviewing students’ written chart-paper solutions; observing the student pairs’ presentations; and hearing other students’ paraphrasing and questioning. Her observations of the students’ work and their reasoning also allowed her to select and sequence the presentations so that students could make connections between the various strategies and concepts. Math Forum gives Angela a strong sense of individual students’ as well as the whole class’s understanding of mathematical concepts.

**CASE STUDY 2: INTERACTIVE QUizzes**

Claire, who teaches grade 9 mathematics, has developed some unique quizzes that play a key role in her assessment plan. She uses both paper-and-pencil quizzes and interactive “clicker” quizzes for formative assessment. (Clickers are a handheld electronic response device.) The quizzes tell both teacher and students of students’ progress, Claire reports: “One of the strategies I use is a quiz …

[students] usually get two or three a week for every unit … they keep their own records, and they do their own assessment of it … they have a quiz book—it’s like their quiz journal.”

During paper-and-pencil quizzes, Claire allows students to ask for assistance from her or from peers and to use their notes. She records what her students are able to do with and without support. These notes and the students’ performance on the quizzes become the basis for conversations with students about their understanding. Claire states: “Most of them do well on the quiz because they get a lot of support … then as we get into the unit, I can tell them, ‘If you are still asking me for help, you’re not going to be ready for your test.’ And that’s a signal that they have to come in [for help] after school. So I get a lot of mileage out of those quizzes upfront.”

Claire also uses clicker quizzes at the beginning of some lessons to generate discussion and get a sense of what students already understand about a topic. In one lesson, Claire was introducing the topic of lines of best fit, moving toward the concept of slope. However, she first wanted to review and consolidate students’ sense of correlations so that she could connect these new ideas to their prior knowledge.

Claire began the class by presenting several multiple-choice questions on correlations, posting them one at a time on the interactive whiteboard (see fig. 3). Students were given a few minutes to work on the question individually, and then each student chose a response using his or her clicker. A circle graph displaying the range of student responses for the class was then shown on the interactive whiteboard. Student responses also became part of a database that Claire could access later to review individual responses. The circle graph showed that the class had chosen each of the four alternatives about equally.

Students were then encouraged to work with a partner to discuss the question. After the pairs talked, Claire asked students to respond individually to the question again. The responses were now more closely aligned with one another as well as with the correct response. Claire then led a discussion about why that response was correct and why students
picked the incorrect responses. Some students stated that they picked solution C first because they were confused about which was positive and which was negative. Many stated that they would rule out D because the data were “all over the place.”

In her interview, Claire described this activity as a way to check students’ understanding. Although students enjoy using the clickers, occasionally she uses a similar strategy without the clickers: She has students write their response on small individual whiteboards that they hold up so that she can see their responses. She then has them discuss their responses in pairs and respond again. Thus, even without the clickers, Claire gets a sense of students’ understanding, and the class still engages in discussion and argumentation, both of which she thinks are vital to developing students’ mathematical reasoning.

CASE STUDY 3: QUESTIONING, LISTENING, AND RESPONDING IN WHOLE-CLASS DISCUSSIONS

Terry, a grade 10 teacher, integrates formative assessment within her lessons, particularly in the way in which she questions, listens, and responds to student thinking. This approach is an important component in classrooms in which students are engaged in problem solving and investigation, but it is not always evident in the structure of secondary school mathematics lessons. We observed Terry’s grade 10 class over several days as students used their prior knowledge of linear relations to investigate quadratic relations.

Terry began one lesson by modeling a linear relationship with stacked linking cubes. She made a series of stacks of cubes and then drew the associated table of values. The first tower had 4 cubes, the second had 7 cubes, the third had 10 cubes, and the fourth had 13 cubes. She asked, “What kind of relation is this?” After students had offered responses, Terry encouraged them to offer more information about the relationship and their understanding of it by asking questions such as, “Why is it linear?” “How do you know?” and “If you were in grade 3 or grade 4 and I asked, ‘Why is this a linear relationship?’ and you didn’t know anything about slope or first differences or anything, how would you explain it, do you think?”

Students then worked in groups to construct models of a linear function. Terry assigned a first difference to each group, and the groups worked with linking cubes to construct the models. After several minutes, the groups shared their models, and a whole-class discussion of the models ensued. To promote discussion, Terry asked questions such as, “Is there anything else that you notice?” She also prompted students to listen to other students’ responses to encourage their reasoning and expand their understanding of particular concepts. Groups then compared models and equations. Two groups, each given a first difference of 3, had different models; one group modeled the equation $y = 3x$ and the other modeled the equation $y = 3x + 4$.

To make the transition to quadratic relations, Terry then presented color-coded linking-cube models of quadratic relationships that she had prepared before class. The color coding helped show the differences between each successive step of the relation and gave students a sense of the curved nature of this relationship. Figure 4 shows an example of a model of a quadratic relationship and the color-coded first and second differences. Terry then worked with the class to create a table of values for the model, and students determined and discussed the characteristics of the relationship.

Groups were then asked to create a quadratic relationship with specific criteria using different colored blocks. The groups worked in several ways, and Terry asked them to “be prepared to explain how you figured it out.” When all the groups had brought their models to the front, Terry used questioning to discuss the models as a class, and then students used graphing calculators to represent their quadratic model graphically. They compared the linking-cube model, the table of values, and the graphical representation for each quadratic. Terry asked, “What does it mean that those two models are equal? What is the same?”

Terry then led a discussion about some characteristics of quadratic relationships. She asked...
students whether they noticed any patterns between the equations and the first and second differences of each table. Several students recognized that the coefficient of $x^2$ appeared to be half the second difference. At least one student also noticed that the way in which the graph opened depended on the sign of the coefficient of $x^2$. As the class ended, Terry asked students to come up with some conjectures about other properties of quadratics and to be prepared to share them the next day.

Throughout this lesson, Terry encouraged students not only to consider what they were doing and how they were doing it but also, by thinking about what makes a relation linear or quadratic, to question why. Terry claimed that her questioning provided feedback to the students; in addition, she learned a great deal about her students’ prior knowledge, misconceptions, and current understanding of mathematical ideas. Terry also used students’ responses to adjust her pedagogical strategy.

CONCLUSION
Incorporating new and varied forms of assessment helps make mathematical reasoning and sense making visible to both students and teachers. Walking around the room while students solved problems helped Angela see students’ reasoning and how they connected mathematical ideas. It also helped inform her selection and sequencing of the presentation of students’ solutions. These presentations made the students’ varied strategies and reasoning visible to the entire group, allowing students to make connections to and between the various mathematical ideas presented. Claire’s use of clicker quizzes and discussion of responses brought student reasoning to the forefront. Terry’s questions sought more than just correct answers; they prompted students to reason and make mathematical sense about the similarities and differences between relationships.

These three teachers’ practices provide clear examples of ways to incorporate formative assessment both to generate and view students’ mathematical reasoning and sense making and to inform next steps.

REFERENCES