If we believe that doing mathematics is an act of sense-making; if we believe that mathematics is often a hands-on, empirical activity; if we believe that mathematical communication is important; if we believe that the mathematical community grapples with serious mathematical problems collaboratively, making tentative explanations of these phenomena, and then cycling back through those explanations (including definitions and postulates); if we believe that learning mathematics is empowering and that there is a mathematical way of thinking that has value and power, then our classroom practices must reflect these beliefs. Hence, we must work to construct learning environments in which students actively engage in the science of mathematical sense-making.  
(Shulman, 1994, p.60).

AIMS AND OVERVIEW

This course examines content and methods for teaching mathematics to Middle and High School students. The philosophy and content of this course reflect the National Council of Teachers of Mathematics Focus in High School Mathematics – Reasoning and Sense Making, as well as the Massachusetts Mathematics Curriculum Frameworks.

It is increasingly accepted that “teacher effects on students’ achievement are driven by teachers’ ability to understand and use subject-matter knowledge to carry out the tasks of teaching” (Hill, Rowan, & Ball, 2005). This class will help you develop the ability to understand the foundations of the mathematics you will teach. We will focus on the salient mathematical points, explore and deepen our understanding of subject matter knowledge for teaching, and consistently explore strategies to formatively assess student learning. Our discussions around a single rich task will focus attention not only on the connections between different approaches, but lead to discourse about the mathematical structure of the tasks.

This is a course in problem solving. It cannot be over-emphasized that there is a difference between solving problems and problem solving. One of the purposes of this course is to alert you to
this difference and, hopefully, to give you some appreciation for the difference. This difference should become increasingly apparent as the course progresses.

Good problems that can be used in the context of a classroom setting to illustrate the problem solving process are not easy to find. The problems we will examine in this class have been selected to help you to discover new concepts and relations, hopefully helping you to gain a better understanding of the interrelation of some mathematical problems and ideas. I am not looking for answers to the problems given. The answers are already known. I am hoping that you will expand on the answers obtained, look for alternatives to the solutions, and for new avenues for investigation. As you move through these experiences, you will be encouraged to reflect on the processes you use, to gain insight and to begin to consider ways to help your students understand.

The hope for this course is for you to participate actively in a creative process, that you be creative in formulating problems and solutions. As problems today become more varied and more complex, increasing interest is being directed at the process, and attempts are being made to systematize and structure problem solving. The problem is that this effort to systematize may run counter to the problem solving process itself. If the process becomes too structured, the creativity and inventiveness essential to the problem solving process will be lost. Things that bring too close a structure, or too tight a system, limit the essential creativity and limits or even destroys the problem solving process itself. There must be room for many different problem-solving processes, just as there are many different problem solvers each exercising their own individuality and creativity in problem solving.

While there may not be any one problem solving process, there are aspects common to each of the various processes, and we can profit by considering some of these common aspects. This can best be done in the context of solving problems.

There are two major professional goals of the this class: (1) to build an awareness of the learning process, the meaning of understanding in mathematics and how it relates to instruction, and (2) to become familiar with the resources and to build skills necessary to plan effective, standards-based math lessons and how to adapt lessons to maximize opportunities for students’ learning. These goals can best be achieved if the prospective teacher:

1. Explores basic mathematical concepts and procedures in number sense, measurement, geometric sense, probability and statistics, and rational and real numbers
2. Applies major principles of learning theory when designing experiences in elementary school mathematics
3. Identifies and chooses among alternative resources and strategies for reaching specific objectives
4. Becomes familiar with professional associations and publications that pertain to the teaching of mathematics, such as the curriculum, evaluation, and assessment recommendations of National Council for Teachers of Mathematics (NCTM), and the Massachusetts State Mathematics Frameworks
5. Experiences technology as a tool for exploring mathematics

**Modeling Real-World Problems**

Much of the mathematics you have done in the past was probably based on things you would never see in the “real” world. In this class, we will try to provide authentic models for the mathematics. The procedure has five stages:
1) Understanding the problem – List what is given in plain terms and list the questions that need to be answered.
2) Identify the quantities that are relevant to the situation – Give letter names to each item that varies and to unknown constants. Usually drawing a picture is helpful.
3) Express in mathematical terms the relations that hold among the variables and the questions that have to be answered.
4) Solve the mathematical relations from 3), keeping in mind the problem to be answered.
5) Interpret the results of the calculations in the language of the original situation.

**Required Text:**
ISBN 978-87353-677-6
*5 Practices for Orchestrating Productive Mathematical Discussions*
Smith, M. & Stein, M.
National Council of Teachers of Mathematics

**ADDITIONAL READINGS WILL BE ASSIGNED AND WILL BE AVAILABLE ON THE COURSE WIKI**

**Recommended Resources**
*Massachusetts Mathematics Curriculum Framework*
http://www.doe.mass.edu/frameworks/math/2000/toc.html


You are encouraged to apply for membership in NCTM. Membership information is available on the NCTM website at http://www.nctm.org.

*Common Core State Standards Initiative.* http://www.corestandards.org/the-standards/mathematics

Additional readings will be emailed or made available online.

**GRADING POLICY**
Your grade will be based on a percentage of the total points possible for the term:

- 100% – 94% = **A**
- 93.9% – 90% = **A**-
- 92.9% – 89% = **B**+
- 89.9% – 87% = **B**
- 86.9% – 83% = **B**
- 82.9% – 80% = **B**-
- 79.9% – 77% = **C**+
- 76.9% – 73% = **C**
- < 73% = **F**
COURSE REQUIREMENTS (Complete descriptions follow)

| Attendance and In-class Assignments (25%) | Assignments will be completed both in class as well as homework. Please keep in mind that attendance in critical. If you must be absent for all or part of a class, it is your responsibility to let Mike know as far in advance as possible. |
| Homework and Task Rewrites (15%) | Homework will be assigned in each class and will be due the following class. There will be one Task Rewrite assignment each week beginning week 2. |
| Take Home Midterm (10%) | Hand out October 29, due November 5. |
| Mini-Teach Project (10%) | Teach a short lesson to 2 students. Proposal due September 24. Teaching Report due October 22. |
| Learning Segment Plan (30%) | For the Learning Segment, you will describe and plan a series of 3–5 mathematics lessons. This will include lesson plans, instructional materials, and assessment tools/procedures. Draft #1 due October 15; Draft #2 and Peer Review November 19; Plan due December 10. |
| Take Home Final (10%) | Due no later than 4:00 PM on December 17. |

Email protocols: When emailing Mike with questions or to submit work, please put **EDCG 669** at the beginning of the subject line.

A note on written assignments: In order to facilitate task debriefs and discussions, as well as timely grading and return of your work, a hard copy of the mathematics homework assignments will need to be handed in at the beginning of each class. A soft copy of the Task Rewrites, Midterm, Mini-Teach Report, Final, and Learning Segment assignments should be emailed to Mike before class on the due dates (a hard copy for these assignments will not be necessary).

Attendance: What we do in class is an essential building block for your learning. Communicating and understanding your own and each other’s reasoning can only happen here. Regular attendance is an indispensable element of classroom participation, so please keep in mind that attendance and active participation is critical. If you must be absent for all or part of a class, it is your responsibility to let me know as far in advance as possible. The make-up policy for absences is detailed below.

Tentative Class Organization: We will start promptly at 4:00 every class day. A portion of some classes will be set-aside for you to work in small groups. The composition of the groups will be up to the instructor.

Homework: Homework assignments with selected problems and reading assignments will be posted on the class wiki. Homework problems should be thought of as equivalent to a take-home test. You have access to resources, notes, and friends. The product turned in should be first class. If you
are disappointed in the score received, problems may be redone and resubmitted (stapled to original work) within seven days.

**Remember:** Doing all homework problems will not necessarily guarantee success, but not doing the work will generally guarantee failure.

**Math Task Rewrite** - What are the characteristics of a “good” task? You will rewrite a task two times; first, to raise the cognitive demand of the task, and second, to lower it. You will also provide a short explanation of the rationale used in the rewrites. The focus of this activity is to take an explicit look at the characteristics of the mathematics tasks given in classes. Tasks which only require students to remember routine procedures will lead to reduced opportunities for student learning. The goal of mathematics instruction is to have students make purposeful connections between mathematical concepts. Tasks that lead to high cognitive demand afford students a chance to think deeply about the concepts underlying the mathematics, require students to explain and justify answers, and allow multiple solution approaches.

**Mini-Teach Project** – This involves teaching on a small scale. You are to choose something from class and teach it to a pair of learners. Ideally, they should be children at your pre-practicum school, but if you have no access to children, two adults are also fine. You will decide what you plan to do, and write up a page or two telling me:

1) What mathematical concept you intend to teach
2) Your subject audience
3) What activity or problem you are going to use
4) How you plan to present the activity or problem
5) What you expect the learners to get out of the process.

You will turn this in to me by September 24. I will check it over and either OK it or make suggestions about possible changes. These should be word-processed, 12-point font, double-spaced.

You will carry out your proposed lesson and report on the result, noting how it compares with your expectation, and observing and reporting how the learning process of your students compares and contrasts with your own. I am very interested in how you interpret your student's thinking. Try to be clear in your explanations. The follow-up report, together with the original proposal, will be due October 22.

**Learning Segment Plan** – The focus of this assignment will be on the purposeful sequencing of mathematics tasks and on the assessment and evaluation of student learning. For the Learning Segment, you will describe, plan, and create lessons and an assessment plan for a series of 3–5 mathematics lessons. A learning segment prepared for this class should reflect a balanced approach to mathematics, including opportunities for students to develop conceptual understanding, procedural and language fluency, mathematical reasoning/problem solving skills, as well as to communicate precisely. This assignment is modeled on the *Teaching Performance Assessment* (TPA) that is being designed by a consortium of 17 states as a way to “level the playing field” for preservice teacher education by providing for a similar, coherent experience. The TPA standards are based on current research and the *National Board for Professional Teaching Standards* (NBPTS).
The assignment is comprised of three parts:

- **Part A: Context for Learning**
- **Part B: Lesson Plans**
  - Lesson plans including assessment tools/procedures
  - Instructional materials
- **Part C: Planning Commentary**

Full descriptions, directions, and rubrics are posted on the wiki. An electronic copy of the drafts and final product are to be emailed to Mike prior to the start of class on the due date. Any worksheets or pages from the student textbook should be scanned and emailed with the documents.

In order to provide you with the time and support necessary to successfully complete the project, the components will be scaffolded according to the following schedule. Mike will provide notes for each of two drafts, and a peer review session will also be conducted in class so that you may learn from and provide feedback to your classmates.

- **October 15:** *Draft #1 due.* Please submit at minimum a first draft (it can be a rough draft) of the Lesson Plans and Cover Page. Full points will be awarded for all drafts submitted. Failure to submit drafts on time will result in a 10% penalty on the final submission.
- **November 19:** *Draft #2 due.* This draft should include all three parts. Full points will be awarded for all drafts submitted. Failure to submit drafts on time will result in a 10% penalty on the final submission. We will also have a Peer Review Session on this day.
- **December 10:** *Learning Segment due.* This is the last class day. Please email the final product to Mike prior to class.

**Templates** for all of the Learning Segment components will be discussed in class and are available on the wiki site.

The grading rubrics for the Learning Segment are also on the wiki. The rubrics use a 5 point scale (1 – 5, with 1 and 2 indicating below standard, 3 represents meeting standard, 4 should be considered A or A-, and 5 would be awarded for truly exemplary work). Please read them carefully and use them for guidance, as this assignment is a major part of your grade.

**Make-up Assignment: Annotated Bibliography.** If you miss a class, the following procedure must be followed to make up for the points lost. Review a contemporary article from the periodic literature that is related to the topic covered on the day you missed. Use the style and format listed in the Annotated Bibliography section below. Turn in the review along with a note describing the date and reason for your absence. As a Make-up Assignment, the article submission is due one week after the absence. Contributions are to be emailed to mike.gilbert@umb.edu

1. Articles must be selected from a professional journal (Examples are listed below. Some are available in print form at the library, many are available online).
2. Write a brief one or two paragraph summary of the article suitable to keep on a file for future reference.
3. Include a further paragraph in which you make your own response to the article. Is this information valuable to an elementary school teacher? Does it include activities you could use in your classroom? What are the important mathematical concepts discussed?
4. These reports will be posted on the course website as a resource for the rest of the class.
5. Identify the article using APA style reference heading such as:


List of acceptable publications:
- Teaching Children Mathematics
- Mathematics Teaching in the Middle School
- Mathematics Teacher
- Journal for Research in Mathematics Education
- School Science and Mathematics
- NCTM Yearbooks
- Teaching Pre K-8
- Elementary School Journal
- Learning
- Focus on Learning
- Phi Delta Kappan
- You may use other publications, but please check with me first

Please Note: The University strives to make academic accommodations for students with identified special needs. Students with disabilities must register with the UMass Boston Student Support Services office if they require special accommodations at 617.287.5820 or URL: http://www.studentsupportservices.umb.edu/

If you need a little refresher for some math skills, you might check out the Khan Academy website at http://www.khanacademy.org/math/arithmetic/#math/arithmetic?k They have a number of short (3 – 5 minute), free videos that provide a clear explanation of the procedures. They don’t really get into the underlying concepts and the why questions (but that’s what we will do in class), but they could help fill in some gaps. When you do use the Khan materials, please let me know if they helped.
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<th>Due dates and assignment check list</th>
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<tr>
<td><strong>Major Assignment Due Dates</strong>&lt;br&gt;<em>(Please note: Homework will be assigned every week and Task Rewrites start week 2)</em></td>
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<tr>
<td>1</td>
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Reading Assignments and homework are due the following week (Subject to change. Will be posted to wiki)

<table>
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Lesson Implementation Guide (excerpt from *Beyond Good Teaching* Chapter 9).  
Learning Segment Documents |
| 2 September 17 | Lesson Planning Guide (excerpt from *Beyond Good Teaching* Chapter 9)  
Handouts for Mathematical Notations (excerpts from *Beyond Good Teaching* Chapter 2) |
| 3 September 24 | *Seven Keys to Effective Feedback*  
*Assessment Overview* Powerpoint  
*5 Practices* Chapter 1 |
*5 Practices* Chapter 2 |
| 5 October 8  | *5 Practices* Chapter 3 |
*5 Practices* Chapter 4 |
| 7 October 22 | *5 Practices* Chapter 5 |
| 8 October 29 | *5 Practices* Chapter 6 |
| 9 November 5  | *5 Practices* Chapter 7 |
| November 12  | *5 Practices* Chapter 8 |
*5 Practices* Chapter 8 |
| 11 November 26 |  |
| 12 December 3 |  |
| 13 December 10 |  |